Empirical labor search: A survey

Zvi Eckstein[a,b], Gerard J. van den Berg[c,*]

[a]*Tel Aviv University, Israel*
[b]*Department of Economics, University of Minnesota and CEPR; 271 19th Avenue SE,*
Minneapolis, MN 55455, USA
[c]*Department of Economics, Free University Amsterdam, Netherlands and IFAU, IFS, IZA, CREST, CEPR*

Abstract

This paper surveys the existing empirical research that uses search theory to empirically analyze labor supply questions in a structural framework, using data on individual labor market transitions and durations, wages, and individual characteristics. The starting points of the literature are the Mincerian earnings function, Heckman's classic selection model, and dynamic optimization theory. We develop a general framework for the labor market where the search for a job involves dynamic decision making under uncertainty. It can be specialized to be in agreement with most published research using labor search models. We discuss estimation, policy evaluation with the estimated model, equilibrium model versions, and the decomposition of wage variation into factors due to heterogeneity of various model determinants as well as search frictions themselves. We summarize the main empirical conclusions.
© 2006 Published by Elsevier B.V.

1. Introduction

This paper surveys the existing empirical research that uses search theory to empirically analyze labor supply questions in a structural framework, using data on individual labor market transitions and durations, wages, and individual characteristics. This literature has

---

*Corresponding author.

E-mail addresses: eckstein@post.tau.ac.il (Z. Eckstein), gjvdberg@xs4all.nl (G.J. van den Berg).

analyzed variety of issues, such as the return to schooling, the interpretation of race earnings differentials, the duration of unemployment and the wage dispersion for homogeneous workers, the effect of the minimum wage, training programs, aggregate labor market unemployment and earnings inequality.

The literature, which started off in the early 1980s, builds on the estimation of Mincerian earnings functions, Heckman's classic selection model, and dynamic optimization theory. Earnings functions express earnings in terms of variables like work experience, which may be determined simultaneously with earnings, so that they are potentially endogenous. The classic Heckman selection model provides a framework to handle this endogeneity problem in a static (or long run) setting.[1] Dynamic optimization theory provides a unified framework to simultaneously explain short term and long run outcomes of earnings, wages, and time out of work. Search theory, in particular, is useful for this purpose. It builds on the principle that it takes time to discover acceptable trading opportunities in the labor market. The imperfect information that is involved is usually denoted by the phrase "search frictions".

A second motivation for the literature concerns the persistently high unemployment levels in Europe and elsewhere. Traditional neoclassical labor market models are unable to explain long spells of possibly involuntary unemployment. Search models have proven to be fruitful tools for the understanding of unemployment durations and the effectiveness of labor market policies aimed at bringing the unemployed back to work (see e.g. the survey by Mortensen and Pissarides, 1999).

Search theory postulates that certain events (like wage offers) occur randomly from the point of view of the individual. This generates probability distributions for observed labor market outcomes like unemployment and job durations and wages. Data on individual labor market outcomes can then be used to estimate the structural parameters of search models. This solves the dynamic selection effects. Moreover, with structural inference the model that is estimated is formally consistent with the underlying economic "story", because one estimates the determinants of the agents' decision problems. Structural empirical inference also enables a formal test of the adequacy of the theory, and it allows for a careful assessment of the effects of policy changes that is not subject to the Marschak–Lucas critique. Contrary to many popular evaluation methods, it allows for an evaluation of counterfactual policies, that is, policies that have not been implemented in reality.

In this paper, we start with a general framework for the labor market where the search for a job is a random sequential process. Following the search literature we show how the general model can be specialized to be in agreement with most published research using labor search models. Specifically, we show how this framework can be restricted to include the following theoretical models: the standard "classical" search model of Mortensen; the search-matching-bargaining model of Diamond and Maskin (1979), Mortensen (1982) and Pissarides (1979); and the wage-posting equilibrium search models popularized by, notably, Albrecht and Axell (1984) and Burdett and Mortensen (1998). For all these models we present the basic properties of the (equilibrium) solution and we explain how one can derive from the model the joint distribution of observations on unemployment durations, accepted wages, etc. We then present the likelihood function of the model

---

[1]Rosenzweig and Wolpin (2000) argue that one cannot hope that there are enough instruments to control all the selection bias.

parameters based on standard panel data of individuals, and we discuss identification and estimation of the parameters of these models.

The search strategy of the firm, the worker and the equilibrium wage determination process vary greatly across different models, due to differences in the assumptions made in these models. Yet, all these models generate a dynamic selection process of work that has important implications on the way we empirically interpret labor market data of unemployment and employment durations and observed wages of workers. The survey makes the following main points:

1. All the models have the same common basic structure. The only real difference between the models concerns the equilibrium concept and the wage strategies that workers and firms are assumed to be using. The equilibrium concepts cannot be justified by the nature of the economic environment but are either taken as given or based on assumptions concerning information and strategies.

2. With only data on unemployment durations, wages and individual characteristics, the joint likelihood function of the parameters of the model has a common format with common identification and estimation problems.

3. Several important economic implications have been derived from the applications regarding labor supply and individual behavior. First, the results allow for various informative decompositions of individual labor market outcomes. Wage variation can be decomposed into separate effects of individual heterogeneity, firm heterogeneity, and search frictions. The exit rate out of unemployment and the mean unemployment duration can be decomposed into choice (voluntary) and chance (involuntary) components. Secondly, the results enable the quantitative evaluation of labor market policies that affect worker and firm behavior, like unemployment compensation (insurance), minimum wages, and taxes.

4. The main advantage of the analysis of equilibrium models is that it enables us to measure the implications of the inefficiency due to the market structure with imperfect information.

During the past decades, some surveys have been published on related topics. Eckstein and Wolpin (1989a) examine the specification and structural estimation of dynamic discrete choice models, paying special attention to computational issues. Wolpin (1995) describes the structural estimation of partial job search models. Rogerson et al. (2005) present a comprehensive introductory up-to-date account of job search theory. Mortensen and Pissarides (1999) survey recent developments in the theoretical analysis of equilibrium search and matching models. van den Berg (1999) surveys the literature on the empirical analysis of equilibrium search models. In comparison, we focus on the common theoretical and empirical structure of different approaches, and on the actual results. Also, we update the state of the art, and we assess the empirical achievements of the search approach as a method to understand various labor market issues.

The outline of the survey is as follows. In Section 2, we present the general model framework. In Sections 3–6, this is specialized to, respectively, classical job search models, on-the-job search models, equilibrium matching/bargaining models, and equilibrium search models with wage posting and heterogeneous agents. Section 7 concludes.[2]

Sections 3–6 have a common structure: after a presentation of the theoretical framework, we discuss the empirical implementation and estimation strategies, and we

---

[2]The so-called Phelps Volume (Phelps et al., 1970) is widely seen as marking the onset of the widespread use of search theory in economics. As far back as 1962, Karlin (1962) analyzed a search model for asset trade with discounting that is very similar to the standard sequential job search model.

subsequently pay attention to the main empirical and policy findings in the corresponding literature. It should be emphasized that the dichotomy between theory and empirics (and the suggested dominance of theory over empirics) becomes less and less accurate the closer we get to the present state of affairs. Flinn and Heckman (1982), Eckstein and Wolpin (1989) and the subsequent literature have adjusted theoretical models to account for empirical regularities in the data used to estimate their models. In fact, current theoretical developments are driven to a substantial extent by the need to explain regularities found in the empirical literature. In particular, many equilibrium search models with heterogeneity have been developed in response to the empirical studies that were unable to explain important findings with homogeneous models.

2. A general framework

We specify a model that is much more general than any existing search model. The main reason is to show that a wide range of theoretical and empirical research in labor economics has a common basic structure. The model therefore also allows us to make a connection between different sorts of results. By specializing the general model, it can be tailored for specific empirical research. In addition, the general framework reveals the limitations of the existing models, and it provides directions for future work.

2.1. Production[3]

Each worker is assumed to provide either zero or one unit of labor. In general, let p_{ij} be the per period unit of labor output by a worker i in firm j. We assume that $p_{ij} = p(x_i, z_j)$ where x_i is a vector of the worker i labor skills (e.g., education and experience) and z_j is a vector of the firm j production skills (e.g., capital stock and production efficiency). Output of firm j is denoted by y_j and is given by

$$y_j = \sum_{i=1}^{I} p(x_i, z_j) \, l_{ij} \tag{1}$$

where I indicates the number of types of workers and l_{ij} denotes the number of workers of type i at firm j. The workers are ordered such that for all j, $p_{Ij} > p_{I-1j} > \cdots > p_{1j}$.[4]

In Section 4, we further generalize this production technology, by allowing for a match-specific component that may have a non-degenerate distribution for given i, j.

2.2. Offer arrival (matching)

Unemployed and employed workers of type i sample job offers from firm j sequentially, at Poisson rates $\lambda_{i,u}$ and $\lambda_{i,e}$ while unemployed and employed, respectively. These Poisson rates vary over i (or, equivalently, over x_i) and can in general be thought to depend on the

---

[3] We avoid explicit definitions that separate between discrete and continuous cases, while we actually specify continuous time models.

[4] The simple additive linear production function is used in most search equilibrium models. The discrete productivity order is used in few papers that solve for the equilibrium wages (e.g., Albrecht and Axell, 1984; Eckstein and Wolpin, 1990). One can also consider a continuous productivity difference, but this is exactly what z accounts for in the value of p.

distribution of firms, on search efforts made by unemployed and employed workers and firms, and on the magnitude and composition of aggregate unemployment U and the aggregate amount of vacancies V in the market.

2.3. Workers

There is a measure M_i of workers of type i, who face a mortality rate η. In each period there are ηM_i newborn workers. Layoffs occur at a rate δ. When a worker of type i with skills x_i is unemployed she/he receives a flow of leisure and, possibly, unemployment benefits. In addition, there is a flow cost of search effort when the worker is unemployed.[5] The sum of these two flows is equal to the worker utility flow in unemployment, which we denote by u_{it}^u. This notation highlights that all elements in this sum may change over time. When a worker of type i with skills x_i is employed she/he receives a flow of earnings of w_{it}. This minus the flow cost of search effort equals the instantaneous utility in the state of employment $u_{it}^e(w_{it})$. Let $d_{it} = 1$ if the individual is working and $d_{it} = 0$ if the individual is unemployed. The worker aims to maximize the expected lifetime utility. At $t = 0$, in the state of unemployment, this present value can be expressed as

$$V_0^u = \mathrm{E}\left[\int_0^\infty \mathrm{e}^{-(\mu+\rho)t}(u_{it}^e(w_{it})d_{it} + u_{it}^u(1 - d_{it}))\,\mathrm{d}t|\Omega_0\right], \tag{2}$$

where the expectations are taken over all the random variables in the model, Ω_0 is the information available to the worker at time $t = 0$, and ρ is the discount rate. In some models workers are assumed to be price takers and in others they bargain over the wage with firms.

2.4. Firms

The profit flow of firm j is given by

$$\pi_j = \sum_{i=1}^{I} (p(x_i, z_j) - w_{ij})\, l_{ij}(w_{ij}), \tag{3}$$

where w_{ij} is the wage that firm j pays to worker i, and where we highlight the dependence of l_{ij} on this wage. We assume that w_{ij} is constant over time. Firms are assumed to maximize the expected present value of profits. However, the firms' set of admissible strategies differs across the models that we survey below.

2.5. Equilibrium wages

Three main equilibrium approaches are used in the literature. The first assumes that the wage offer distribution equals the distribution of the worker's marginal (average)

---

[5]In many models in the literature, search costs and search effort, for unemployed and employed workers, are optimally determined by equating the marginal costs and benefits of effort. See e.g. Mortensen (1986). We abstract from these aspects of the models since the empirical work on this is relatively small. However, we think that the economic insights and the empirical potential of understanding the endogenous determination of the search frictions is of great potential for the empirical research. In terms of a general framework the search efforts on and off the job are certainly part of the existing search theory.

productivity, that is, $p(x_i, z_j) = w_{ij}$. This formulation follows Lucas and Prescott (1974) where the wage offer distribution is equal to productivity that is different across locations ("islands") populated by different firms.[6] One may call this a "competitive equilibrium" since profits are zero. With exogenous productivities, the wage offer distribution in this approach is effectively exogenous. Now, consider partial job search models. These by definition have an exogenous wage offer distribution. Such models can be given an equilibrium interpretation by thinking of wages as being equal to productivities. Along these lines, existing empirical analysis with partial models can be justified as equilibrium analyses.

However, suppose that unlike the assumption of Lucas and Prescott, firms "on the same island" can costlessly offer a wage different from the worker marginal product. Now firms can typically improve on their profits by offering a lower wage than the marginal product and no firm would respond. In general, due to the presence of search frictions firms have monopsony power that now they can use in equilibrium. For a worker matched to a firm, it takes time, effort, and, ultimately, money to find an alternative match. Because of this, a match may carry a positive rent or surplus. If a match is dissolved then such a surplus evaporates. Agreeing to form a match involves agreeing on how to divide the surplus over the two parties. The firm's part of the surplus can be taken by paying a wage below the marginal product. In sum, the worker does not refuse all wage offers below the marginal product.

The above point is a well-known argument that was made by Diamond (1971) as a critique of the simple search model. That is, with one reservation wage the firms can take the entire rent by offering to the worker his/her reservation wage. This point and the observation that there is a large variance in actual wages for workers with the same observed quality (there is no "one price for the same product") have led the theoretical literature to consider alternative labor market equilibrium and information setups. The two main concepts are the matching-bargaining model (see Mortensen, 1982; Diamond and Maskin, 1979; Pissarides, 1979, 1990), and the "posting wage equilibrium" where the firms set wages but their information on workers' history is not complete.[7]

The determination of equilibrium wages in the search-matching-bargaining equilibrium models is the outcome of a decentralized bargaining game between the two parties involved in a match. According to the Nash Axiomatic solution to this bargaining game, a fixed fraction of the surplus of the match goes to the worker in the form of a wage. The strategic bargaining approach provides an equivalent framework (see Wolinsky, 1987).

Alternatively, the equilibrium is derived by assuming that the firms have all the bargaining power but they compete by posting wages prior to the moments at which applicants are found (MacMinn, 1980; Burdett and Judd, 1983; Albrecht and Axell, 1984; Mortensen, 1990; Burdett and Mortensen, 1998). Just like in the search-bargaining model, the wage is typically lower than the marginal productivity. Moreover, the wage (distribution) depends on the determinants of the behavior of all agents in the market.

---

[6]See Mortensen's (1986) explicit reformulation of this approach within a search model with search on the job. Mortensen also discusses the limitation of this approach for the description of the equilibrium labor market.

[7]Recently, Rocheteau and Wright (2005) have contrasted these three approaches in monetary economics. For recent developments in the theory of alternative search equilibrium models, see also Rogerson et al. (2005).

3. The job search model

3.1. The estimation of the classical job search model

3.1.1. Theory

Suppose that all workers are homogeneous ($x_i = 1$) and that there is a continuum of firms such that $p(z) = z$ and z has a c.d.f. $F(z), z \in (0, \infty)$. We assume that each firm is atomistic and in equilibrium offers a wage. Following Lucas and Prescott (1974), one may take this wage to equal the marginal product of labor, so that $w = z$.[8] Furthermore, the offer arrival rate when the worker is unemployed is set by $\lambda_{ij,u} = \lambda > 0$, there are no offers while the individual is working, i.e., $\lambda_{ij,e} = 0$, and there are no layoffs and deaths, that is, $\eta = \delta = 0$.[9] Search efforts are exogenous, and therefore the utility in the unemployment state is given by $u_{it}^u = b$. Hence, if an unemployed worker receives a wage offer and she/he accepts it, then she/he will work at that age forever. Now the Bellman equation for the worker's problem can be written as an optimal stopping rule for work, such that

$$\rho V^u = b + \lambda \left\{ \int_0^\infty \max[0, V^e(w) - V^u] \, dF(w) \right\}, \tag{4}$$

where V^u is the maximum expected value of being unemployed and $V^e(w) = \int_0^\infty e^{-\rho t} w \, dt = w/\rho$ the value of accepting an offer w. This equation has the familiar structure of asset flow value equations (see e.g. Pissarides, 1990). The return of the asset V^u in a small interval around t equals the sum of the instantaneous utility flow in this interval, and the expected excess value of finding a job in this interval. When an offer of w arrives at t then there are two options: (i) to reject it (excess value zero), and (ii) to accept it (excess value $w/\rho - V^u$). It is clear that the optimal strategy is to choose option (ii) iff $w > \rho V^u$. Therefore, the optimal strategy of the worker is a reservation wage strategy $w^* = \rho V^u = \rho V^e(w^*)$. Using this condition in the above equation provides us with

$$w^* = b + \frac{\lambda}{\rho} \left\{ \int_{w^*}^\infty [w - w^*] \, dF(w) \right\}. \tag{5}$$

There exists a unique w^* that solves for the optimal reservation wage.

3.1.2. Empirical implications and estimation using duration data

The unemployment duration distribution is fully characterized by its hazard rate as a function of the duration τ of unemployment, $h(\tau)$. In the above model, $h(\tau)$ is a constant h given by

$$h = \lambda(1 - F(w^*)). \tag{6}$$

With a constant hazard rate, the survivor function for τ periods of unemployment is equal to $\Pr(t > \tau) = e^{-h\tau}$, and the density of the unemployment duration τ is given by $g(\tau) = \lambda(1 - F(w^*))e^{-\lambda(1-F(w^*))\tau} = he^{-h\tau}$ (see e.g. van den Berg, 2001). The likelihood function of a

---

[8]The assumption that $w = z$ is inconsequential throughout this section, but it is consistent with the Lucas and Prescott island model where workers in equilibrium receive their marginal product. Alternatively, one may interpret the model as a partial job search model, where the distribution of wages is a structural model determinant. Yet another interpretation is that z is the match value of a firm and a worker and w is equal to a fraction of the match value (see matching models below) that for simplicity is assumed to be one.

[9]The extension to $\delta > 0$ and $\eta > 0$ is immediate.

sample of durations for I unemployed workers, $\{\tau_i, i = 1, \ldots, I\}$, with no incomplete spells of unemployment,[10] is given by,

$$L(b, \lambda, \rho, F(w)|\tau_1, \tau_2, \ldots, \tau_I) = \prod_{i-1}^{I} h e^{-h\tau_i}. \tag{7}$$

Clearly, using only duration data, the only identified parameter is the hazard rate, h, and *none* of the structural parameters of the search model is identified (Flinn and Heckman, 1982). It should also be noted that if the data consist of complete spells, then the hazard rate h can be easily estimated using the sample duration mean, by invoking that $E(\tau) = \int \tau g(\tau) \, d\tau = 1/h$.

3.1.3. Heterogeneity

A common fact is that for a random sample of unemployed workers the Kaplan–Meier maximum likelihood non-parametric estimator of the hazard rate is decreasing.[11] This observation is consistent with the standard stationary search model if we allow for unobserved heterogeneity in parameters of the model. The simplest way to show this is by assuming that there are two types of workers, types A and B, who differ with respect to at least one parameter of the model. Now for each type there will be a different hazard rate. Suppose that $h_A > h_B$, then type A workers leave the unemployment pool on average faster than type B. The pooled hazard rate is the mixture of the two types and converges monotonically to h_B as the duration increases. Now the likelihood function of the same duration sample of I unemployed workers is a mixture of two duration density functions. The separate hazard rates h_A and h_B and the proportions of the two types are identified from the duration data. However, again, *none* of the structural model parameters is identified.

3.1.4. Earnings and job search

Suppose we also have data on the wages that are accepted by workers at the moment they leave unemployment. Specifically, suppose that for each individual who finds a job we observe τ_i and the accepted wage w_i. The joint probability density of the observations is

$$\Pr(\tau_i, w_i) = g(\tau_i) f(w_i | w_i > w^*) = \lambda(1 - F(w^*)) e^{-\lambda(1-F(w^*))\tau_i} \frac{f(w_i)}{(1 - F(w^*))}, \tag{8}$$

together with the restriction $w_i \geqslant w^*$, and with the first term cancelling out with the denominator of the last term.

In labor studies, the wage distribution is traditionally described by Mincer's earnings function: $\ln w = x'\beta + \varepsilon$, where x is a vector of human capital indicators of the worker and $\varepsilon \sim N(0, \sigma^2)$ is a random error that is assumed to be independent of x. In the context of the search model above it is natural to take the distribution of w to represent the distribution F of wage offers, and to take $\varepsilon = \ln z$, that is, the error term in the wage equation is due only to the firm heterogeneity.

---

[10] If the data contain censored observations with duration τ_s then the likelihood of each of these observations is just the survival probability, $e^{-h\tau_s}$.

[11] Eckstein and Wolpin (1995) discussed the evidence regarding this fact. Yet, for certain data sets, the hypothesis that the hazard rate is constant may be accepted.

For the sake of illustration, let the sample consist of initially unemployed individuals with the same human capital characteristics such that $x'\beta = \mu$. The likelihood function conditional on a sample of unemployment durations and accepted wages for I workers, $\{\tau_i, w_i, i = 1, \ldots, I\}$, is now given by

$$L(b, \lambda, \rho, \mu, \sigma | (\tau_1, w_1), \ldots, (\tau_I, w_I)) = \prod_{i=1}^{I} e^{-\lambda(1-\Phi((\ln w^* - \mu)/\sigma))\tau_i} \lambda \frac{1}{w_i} \phi\left(\frac{\ln w_i - \mu}{\sigma}\right), \qquad (9)$$

again with the restriction $w_i \geqslant w^*$, with ϕ the density of the standard normal distribution and Φ its c.d.f.

3.1.5. Identification

For the general case where wage offers are not log-normally distributed, $\Phi(w)$ and $\phi(w)$ in the likelihood function (9) have to be replaced by the nonparametric c.d.f. $F(w)$ and the associated p.d.f. $f(w)$, respectively. The theoretical restriction that $\Pr(w_i < w^*) = 0$ implies that the likelihood is zero if a wage is observed below the reservation wage. In fact, "the minimum of accepted wages (or any fixed order statistic) is a consistent estimator of w^*" (Flinn and Heckman, 1982). By implication, the observable outcomes (accepted wages, durations, and the reservation wage) are not informative on the shape of F below w^*. Indeed, from the above equations it follows that these outcomes are not even informative on the amount of probability mass $F(w^*)$. Intuitively, an individual is indifferent between a high arrival rate with lots of bad offers and a low arrival rate with mostly good offers. Consequently, F and λ are not nonparametrically identified.

More constructively, F is identified as long as some functional form is assumed for F below the reservation wage (and then its parameters can be consistently estimated). It is sufficient if the functional form of F is *recoverable*, i.e. can be recovered from a version that is truncated from below. The family of log-normal distributions satisfies this condition. Of course, if we observe a random sample of offered wages, including wages below w^*, then we are able to identify F nonparametrically. Occasionally, additional information can be used to reduce the dependence of the estimates on the functional form for F. For example, data on numbers or arrival times of job offer enable nonparametric identification of the probability mass $F(w^*)$ which is the job offer rejection probability (Blau and Robins, 1986). van den Berg (1990) uses subjective responses on the mean and upper and lower bounds of the support of F in conjunction with a parametric functional form. Observation of the lower bound in particular is useful: for example, in case of the (nonrecoverable) family of Pareto distributions for F, it reverts nonidentification. It is useful to point out that equilibrium models imply that the wage offer distribution has zero mass at wages below the lowest reservation wage (see Section 6), which in case of homogeneous workers implies that $F(w^*) = 0$.

Identification of F enables us to subsequently identify the job offer arrival rate λ and the job offer acceptance probability $(1 - F(w^*))$ as the two components of the hazard h. This is a central issue in understanding, interpreting and analyzing unemployment data. It enables separation of the two main causes of unemployment, that is, job availability and job acceptance. Moreover, this specification consistently corrects for potential bias in estimating the wage distribution due to the endogeneity of job acceptance. These biases are important when one tries to estimate the mean wage offer for different individuals in order to learn about the return to education, the gains from experience, discrimination, etc.

Note that b and ρ cannot be separately identified. If a value for ρ is set exogenously, as is frequently done in applied micro-studies, then b can be recovered using the estimated reservation wage and the reservation wage equation above. Alternatively, if b is assumed to equal exactly the observed unemployment benefit levels (zero utility of leisure), then ρ can be estimated (see e.g. van den Berg, 1990).[12]

The result that the minimum observed wage is a consistent estimator of the reservation wage puts high demands on the data. A single observation may have a strong effect on the estimated acceptance rate of wages. This ignores the possibility of measurement error in reported wages. In addition, there may be a large small-sample bias. Schoonbroodt (2003) investigates this in detail by way of Monte Carlo simulations for the simple search model. MLE estimation with 500–1000 observations gives rise to a *large* upward small-sample bias of the arrival rate and a small positive bias for the mean wage. The latter is the result of the fact that all observed wages are above the reservation wage and only accepted wages are observed. This causes the MLE to have a large upward bias for the offer arrival rate due to the non-linear relation between these two parameters in the likelihood function. Schoonbroodt (2003) shows that the bias in arrival rates and mean offered wage of moment estimators is much lower than that of MLE since the bias in one parameter does not transfer to a larger bias in the other parameters.[13]

Measurement errors in reported wages can be easily included (Wolpin, 1987) by assuming that $\ln w^{o} = \ln w + u$, where w^{o} is the observed wage and u is a multiplicative measurement error that is independent of the true wage w and is distributed $N(0, \sigma_{u}^{2})$. The likelihood function for this case is the product of $\Pr(\tau_i, w_i^{o})$ ($\neq \Pr(\tau_i, w_i)$) over all the observations.

Heterogeneity can be introduced by assuming that certain parameters are different across different types of workers, as we explained above. Then, the reservation wage is individual specific. Observed heterogeneity can be included by specifying that $\mu_i = x_i'\beta$ as well as making the value of b a function of individual specific observations including benefits. The framework is also easily extended to allow unemployed workers to maximize utility rather than income, taking into account that jobs differ in e.g. hours or non-wage characteristics (see e.g. Khandker, 1988, and other studies mentioned below).

Maximum likelihood estimation can be done fully structurally. For virtually any functional form of F, the reservation wage equation does not allow for an explicit solution, so it has to be solved numerically. The numerical solution is subsequently used as a determinant of the distribution of durations and wages. In sum, the reservation wage equation is solved numerically at each iteration and the likelihood is maximized numerically. Heterogeneity increases computation time, because one needs to solve the reservation wage equation for each type of individual.

3.1.6. Nonstationarity

The above model is stationary because the parameters do not vary over time, and, as a result, the individual hazard rates (h) are constant over time. However, the pool (or

---

[12]If b is a function of observed individual characteristics, then a parametric specification of this function can be identified up to a constant if ρ is not set a priori. Lancaster and Chesher (1983) use subjective responses on reservation wages and expected accepted wages to estimate certain elasticities in the classical search model.

[13]This result provides a good rationale for using moment estimators rather than MLE in case of the potential existence of small-sample bias.

aggregate) hazard rate may be duration dependent due to heterogeneity of the hazard rate among individuals. An alternative and additional source for duration dependence of the hazard rate is that the environment (parameters) at the individual level is not stationary. This includes the important case where the individual parameters change as a function of the elapsed unemployment duration. For example, the amount of unemployment benefits depends on the elapsed duration, or there is a stigma effect on wage offers that depends on the duration of unemployment. Sooner or later these changes are recognized and used by the individual in determining the optimal strategy.

Suppose that the parameters are allowed to vary over the interval of time, $\tau = [0, T]$, in a deterministic way, and job searchers have perfect foresight in the sense that they correctly anticipate changes of these parameters (van den Berg, 1990). By analogy to Section 3.1.1,

$$\rho V^{\mathrm{u}}(\tau) = \frac{\mathrm{d}V^{\mathrm{u}}(\tau)}{\mathrm{d}\tau} + b(\tau) + \lambda(\tau) \cdot \mathrm{E}_{w|\tau} \max\left\{0, \frac{w}{\rho} - V^{\mathrm{u}}(\tau)\right\}. \tag{10}$$

Now, the reservation wage $w^*(\tau)$ depends on τ by way of $w^*(\tau) = \rho V^{\mathrm{u}}(\tau)$. Obviously, the reservation wage solution is more complicated than in Eq. (5). Eq. (6) for h still applies, but now all parameters may vary over time:

$$h(\tau) = \lambda(\tau)(1 - F(w^*(\tau), \tau)). \tag{11}$$

A special case of nonstationarity is a finite horizon search model. Assuming that there is no income after period T, the reservation wage at T equals $w^*(T) = b$, since the option value of search is zero at T. If all structural parameters are constant then it is straightforward to show that the reservation wage is monotonically decreasing before T. That is, $w^*(\tau) > w^*(\tau + \Delta)$ for any $\Delta > 0$. This implies an increasing hazard rate. Incidentally, as shown in e.g. Mortensen (1986), realistic discounting implies that the retirement age horizon does not have a sizeable effect on outcomes of unemployed workers until some years before retirement.

It is straightforward to derive the likelihood function for a nonstationary model using the joint duration and wage data. The likelihood is the product of the joint probabilities of the sequence of available data per individual as in (9). With a hazard rate $h(\tau)$ as a function of unemployment duration τ, the likelihood contribution of a realized unemployment duration τ_i equals $h(\tau_i) \exp(-\int_0^{\tau_i} h(\tau)\,\mathrm{d}\tau)$. Note that the likelihood function now includes the restrictions $w_i \geqslant w^*(\tau_i)$.

The identification issues are similar but not identical. The nonstationary model generates additional identifying restrictions, notably from the reservation wage as a function of the elapsed duration. This implies that the identification may be easier to attain. Yet, the basic Flinn and Heckman (1982) identification problem concerning the wage offer distribution below the reservation wage (see earlier in this Section) applies here as well, albeit that the problem is now the wage offer distribution below the *lowest* reservation wage.

An important advantage from using nonstationary models is that it is possible to solve a search model with an endogenous state space in addition to changing parameters. For example, one may allow for the mean wage and the offer rate to depend on work experience and unemployment duration (see Wolpin, 1987, 1992). In some applications, the initial condition for the differential equation (10) is obtained by assuming that the

value function at time $T + 1$, $V(T + 1)$, is some "ad hoc" specific function of the state variables at time T.[14]

3.1.7. Estimating reduced-form hazard rate models

Consider Eq. (11). It would obviously be useful for any empirical analysis of unemployment durations to be able to separate between the two factors at the right-hand side, i.e. to assess their relative magnitude for different types of individuals, as well as to assess the size of policy effects on them. However, descriptive empirical analyses of unemployment durations motivated by the search model often simply restrict attention to variation of h itself over time and across individuals with different x. A popular approach is to specify h as a multiplicative function of τ and x. This defines the Proportional Hazard (PH) model, which is an ad hoc descriptive specification for h. The Mixed Proportional Hazard (MPH) model extends this by including a multiplicative unobserved heterogeneity term at the individual level (see Lancaster, 1990; van den Berg, 2001, for overviews). In obvious notation,

$$h = h_\tau(\tau) \cdot \exp(x'\beta) \cdot h_{\text{unobserved}}. \tag{12}$$

In the context of job search models, this empirical approach gives rise to some discussion. First, although reduced-form studies interpret estimation results in terms of a job search model, the latter in general does not lead to a "proportional" specification as in the MPH duration model (van den Berg, 2001). According to (11) the hazard rate h at τ depends on all structural parameters in a heavily nonlinear fashion by way of the current reservation wage $w^*(\tau)$. Even if individuals do not anticipate future changes of the structural parameters, and even if the structural parameters are simple functions of τ and/ or x, this leads to a non-proportional expression for h. Because the MPH model parameters are not structural parameters, a causal interpretation of the reduced-form estimates is problematic. For example, from the point of view of the theory, β is not a structural parameter, and a change in one explanatory variable leads to changes in the elements of β that correspond to the other explanatory variables.[15] What is more, a reduced-form analysis cannot separate between the relative magnitudes of the job offer arrival rate and the acceptance probability, or estimate the relative magnitude of the effects of the x variables on them.

A second problem of reduced-form analyses concerns the fact that the effects of explanatory variables are typically identified from inter-individual comparisons. However, differences in, say, the benefit levels across individuals may reflect other differences across individuals, like differences in their past labor market history, and this may not be controlled in a reduced-form analysis. It is therefore useful to look for exogenous variation in the benefit levels, for example in a natural experiment (Meyer, 1990). A third problem concerns instability of parameter estimates of the MPH model. Although the model is identified, the estimates may be very sensitive to functional form assumptions regarding duration dependence and the unobserved heterogeneity distribution. Both the second and

---

[14]The parameters of this function are estimated but it is not necessarily the case that the estimated function is consistent with the lifetime optimal plan (see, for example, Gilleskie, 1998). In van den Berg (1990), the initial condition follows from stationarity of the model after a certain point in time.

[15]To some extent this problem can be captured by allowing for interaction effects, but this may lead to unidentified model specifications. Multi-spell data may be useful in this respect. See van den Berg (2001) for details.

the third problems can be tackled if one has access to multi-spell data, which contain multiple unemployment spells for the same individual. In that case one can exploit variation within individuals to identify the effects of interest. Intuitively, there is an analogy to fixed-effects panel data analysis: if the model specification is invariant across spells then selection effects can be removed by considering differences in individual outcomes across spells.

To sum, the PH and MPH models are simple statistical specifications for a descriptive analysis of duration data in terms of conditional hazard rates. Not much can be concluded regarding potential explanations for the dependence of the duration on exogenous variables and, obviously, it is hard to analyze counterfactual policies using the estimated model.

3.2. Structural estimates of the classical job search model

Kiefer and Neumann (1979) were the first to empirically investigate implications of the above standard search model with constant reservation wages, using reduced form equations. Flinn and Heckman (1982) were the first to structurally estimate the model.[16]

Flinn and Heckman (1982) use a sample of 20–24 years old US white high school graduate males who do not attend school. Unemployment and non-participation are viewed as one state and the individuals are followed since they left school. Using the standard model infinite horizon search model that is presented above with maximum likelihood with two specifications for the wage offer distribution and several other assumptions, their main findings are: (1) The job offer arrival rate (λ) is between 0.13 and 0.20 per month, so that the probability of encountering one (no) offer in the next six months is 0.36 (0.30–0.45). (2) The job acceptance probability is between 0.60 and 0.92 depending on the wage offer distribution function since the reservation wage is estimated by the lowest observed wage. As a result, the estimated mean of the wage offer distribution is lower than the mean of observed accepted wages, and the variance of the observed accepted wages is lower than the estimated variance of the offered wages. (3) The model fits the aggregate unemployment rate for this demographic group in the population from CPS data very well. (4) The results are not robust to the functional form specification of the wage distribution function.

Stern (1989) estimates a continuous time stationary search model where the choice of the individual includes the reservation wage and the number of applications of jobs. The structure of his model is somewhat different than the standard search model presented above, yet the main issues of identification and truncation of the observed wage distribution and the duration distribution of unemployment are part of the analysis. Furthermore, Stern (1989) allows for measurement error in wages following Wolpin (1987), in order to avoid the estimation of the reservation wage by the lowest observed wage. The data are taken from the NLSY79 and the findings are that the job rejection rate is larger than 75% for all males and females in all schooling levels. In addition, the test rejects the hypothesis that workers apply for only one job.

Blau (1991) estimates a model in which instantaneous utility is a weighted product of wage per hour and number of hours, where both are independently sampled when a job offer arrives. Blau allows for measurement errors in wages and hours. Blau uses a sample

---

[16]See, however, Yoon (1981) for structural estimation of a somewhat different job search model.

of 25–35 white high school graduate males who do not attend school and who experienced a spell of unemployment. The estimated weekly offer rate is 0.065 in the first week and 0.037 by the 25th week of search with acceptance rate close to one. The model with hours fits much better the data on duration and wages but the estimated mean weekly offered wage is higher than the observed accepted mean weekly wage, which is opposite to what we expect. This might be due to estimated rate of job rejection being close to zero which is very different than the results of Stern (1989).

The early examples of structural empirical analyses of *nonstationary* models with anticipation are Wolpin (1987) and van den Berg (1990). Wolpin (1987) uses a discrete time search model over finite horizon, and he allowed for the offer rate to go down with the duration of unemployment in order to fit the decreasing hazard observations. Wolpin used a sample of white males who graduated from high school in 1979 (NLSY79) to get their duration of non-employment to first job and the wage at this job. Furthermore, he set the finite horizon to 500 weeks and he introduced measurement error in wages to allow for observed wages to be below the estimated reservation wage. The estimated model fits the non-employment duration data well, due to the estimated decreasing offer probability in order to fit the observed decreasing hazard rate.[17] The results show that the OLS estimator of a log wage equation provides upward selection bias of the constant (conditional mean) as well as in the coefficients of army related psychometric test (AFQT) and father's schooling.[18] Wolpin estimated that the offer rate is about 1% per week and acceptance rate is close to one. As a result, the main impact on unemployment duration is due to changes in the job arrival rates, and there is very small impact for changes in mean offered wages and cost of search. Furthermore, increasing the weekly job arrival rate from 1 to 5% reduces unemployment duration by 60%.

van den Berg (1990) specifies and estimates a continuous time search model where the unemployment benefit levels change for an interval of two years. Using a sample of Dutch unemployed men he estimates that the offer arrival time is between 4% and 6% and the acceptance rate increases from about 70–87% at first data of search to 83–95% after two years. The main findings are that the changes in the benefit levels along the interval of two years and, in particular, the end period level have a large impact on unemployment duration for low and middle level education groups. Because of anticipation, the benefit level after two years has a strong impact on the hazard rate of the short-term unemployed.[19]

The empirical work based on the structure of the simple search model is growing. This model provides the basis for the analysis of active labor market policy reforms that aim at reducing unemployment and increasing participation. For example, Fougère et al. (1998)

---

[17]The finite horizon model implies a decreasing reservation wage, which may fit the data only if a decreasing offer probability or unobserved heterogeneity is also present, which was not part of the estimated model (see Heckman, 1981).

[18]As we explained above, the search model provides a structural selection rule for jobs which correct for potential bias in estimating the conditional mean wage using accepted wages only.

[19]The unpublished paper by Engberg (1991) provides an innovative structural analysis of a nonstationary search model with unobserved heterogeneity. Also, Ferrall (1997) specifies and estimates a discrete time search model where the unemployment benefit levels depend on previous earnings and changes after an interval of time. He uses NLSY79 data for the US and an equivalent data set for Canada. The main findings are that the weekly arrival rates of job offers is about 50%, is increasing by schooling and lower for Canada than the US. The job rejection rates are very close to 90% per week for the US and lower (about 85%) for Canada. The job arrival rates are lower and the job destruction rate is higher for lower schooling workers in both countries.

estimate the classical job search model with endogenous search efforts, focusing on the impact of a public employment service. It is widely believed that the labor supply elasticity at the extensive margins is large. The simple search model provides a useful analytical framework for a quantitative analysis of this elasticity.[20]

4. Search on the job

4.1. Models and empirical implementation

Burdett (1978) has extended the classical search model by allowing employed workers to search further for better jobs after a job has been formed. The model setup is the same as for the classical search model above, with the exception that the job offer arrival rate when employed is positive. We denote this rate by λ_e, and the arrival rate in unemployment by λ_u. In the basic on-the-job search model, a job is characterized by its wage w which is taken to be constant within a job. For a working individual, the search environment is specified in exactly the same way as we did earlier for an unemployed individual. Assume the model to be stationary. The optimal strategy is constant during a job spell, and the expected present value of search $V^e(w)$ when following the optimal strategy in a job with wage w satisfies

$$\rho V^e(w) = w + \lambda_e E_{\widetilde{w}} \max\{0, V^e(\widetilde{w}) - V^e(w)\}, \tag{13}$$

where the expectation is taken with respect to the distribution F of wage offers $\widetilde{w}$ (note that this equation has the same intuitively appealing form as Eq. (5)). Clearly, the optimal strategy is such that one accepts a job if and only if the offered wage exceeds the current wage w, so it suffices to compare instantaneous income flows (i.e., the optimal strategy is "myopic"), and the reservation wage of the employed worker simply equals the current wage. Hence, the worker accepts a job if and only if $\widetilde{w} > w$. For a given current wage w, the hazard of the job duration distribution (or exit rate out of the present job) equals $\theta(w) = \lambda_e(1 - F(w))$. As a result, the duration of a job with a wage w has an exponential distribution with the parameter $\theta(w)$.

If, during employment, exogenous separations occur at a rate δ, then this does not affect the optimal strategy when employed. The exit rate out of the present job then equals $\lambda_e(1 - F(w)) + \delta$. In general, the possibility of search on the job affects the option value of search when unemployed, and the optimal strategy reservation wage becomes[21]

$$w^* = b + (\lambda_u - \lambda_e) \int_{w^*} \frac{1 - F(w)}{\rho + \delta + \lambda_e[1 - F(w)]} \, dw. \tag{14}$$

---

[20]Ljungqvist and Sargent (1998) specify a search model for the unemployed where search effort affects the job offer arrival rate, and the individual faces random death and skill promotion when working. The model's parameters are set to match several moments and other empirically relevant aspects of the economy. The focus is on transitional dynamics of the economy with and without large welfare transfers that reduce labor supply at the extensive margin. An economy with a generous welfare system displays unemployment hysteresis in case of turbulence, because displaced workers have a low search effort at a time when their skills deteriorate.

[21]The derivation of Eq. (14) is by integration by parts using Eqs. (13) and (5). Rogerson et al. (2005) provide an extensive discussion of the derivation of these equations as well as the extension to endogenous arrival rates using the specification where efforts are affecting the arrival rates but require some costs.

Note that if the job offer arrival rates are the same in unemployment and employment then $w^* = b$, and so $h = \lambda_u(1 - F(b))$. In this case the option value of search while unemployed is zero. In general, the reservation wage is lower if search on the job is possible. This model was extended to include endogenous search efforts within the Lucas and Prescott (1974) equilibrium search model (Mortensen, 1986), as pointed out in Section 2.

4.2. Estimating on-the-job search models

The extension of the model to include search on the job and search efforts enable the theory to fully characterize the joint distribution of the observed labor market states: nonemployment (out-of-the-labor force and unemployment), and working in job j during the working cycle c ($j, c = 1, 2, 3, \ldots$). An individual "cycle" is defined by the transition from nonemployment to work and back to nonemployment which is equivalent to the unemployment spell in the simple model.[22] The theory provides the joint distribution for work history for each individual based on the observed states and transitions among these states jointly with the observed wages. The likelihood function is equal to this joint distribution.

The main difficulties are the potential events that the theory predicts have probability zero. As in the simple model, no wage can be observed below the reservation wage and all transitions from job to job should involve a wage rise. The case is that some workers shift jobs with a wage decline. The simplest way to deal with these issues is to follow the idea of measurement error in wages as we described above, following Wolpin (1987).[23]

The identification of on-the-job search models with data on unemployment and job durations and wages is not fundamentally different from the identification of the standard search model (i.e., given a suitable functional form of F, or with the assumption that no unacceptable wage offers are made). In particular, the identification of F, λ_u, ρ and b can be achieved from the data on unemployment durations and accepted wages in the first job in exactly the same way as we discussed it in the standard search model. Now there are two additional parameters λ_e and δ. Given the identification of F, it is possible to identify λ_e from the data moment on the transition rate of workers to new jobs. The moment on the transition rate to unemployment can identify the value of δ. As in the standard search model, ρ and b cannot be jointly identified from the reservation wage without additional information.

The model can be generalized to allow for nonstationarity (see e.g. van den Berg, 2001, for details). The fact that the optimal strategy of employed workers (13) is myopic makes it relatively easy to nest the expression for the job-to-job transition rate into a PH specification. Thus, the relation between theoretical and reduced-form model specifications is simpler than in the case of the standard search model for unemployed workers. This is also true for the exit rate out of unemployment h if $\lambda_u = \lambda_e$. In both cases, the structure

---

[22]It is possible to distinguish between out-of-the-labor force and unemployment within the nonemployment state if search efforts are different at this state. Out-of-the-labor force in the model is the case where the job offer rate from nonemployment is zero (or close to zero). To empirically use this prediction the data should equivalently separate the two nonemployment states.

[23]Flinn (2002a) provides an exact formulation for this likelihood function. We avoid writing it here in order to save on complex details. The issue of incomplete cycles (spells) can be dealt with in the same way as incomplete spells in the simple model. Possible alternative approaches to deal with observed transitions with wage declines are (1) allow for non-wage job characteristics and (2) allow for individuals to anticipate a lay-off.

imposes clear restrictions and interpretations related to the variables that enter the hazard rate.

4.2.1. Search or labor force participation

Eckstein and Wolpin (1989c) analyze the distinction between labor supply theory that is based on the search model vs. the labor supply theory that is based on period by period "new" wage and job draws, which they called "labor force participation model". The search model allows a worker to always stay at the existing job for the same wage as she received at the present period, unless she leaves to another job or quits. In a labor force participation model the worker cannot continue with the same job and wage period after period in certainty. In each period, the worker receives a new job offer as a random draw that is given by a probability that could be equal to one. If the worker receives an offer he draws the wage from a given distribution. The previous wage is then no longer available. Both the offer probabilities and the distribution of offered wages could change due to exogenous as well as endogenous state variables.

There is no aspect of the data that would force us to prefer one of the two models. The papers by Eckstein and Wolpin (1989b), Berkovec and Stern (1991) and Keane and Wolpin (1997), and more recent papers, prefer the labor force participation framework for empirically analyzing dynamic labor supply. The main reason is that since the wage is not part of the state space, the dynamic programming solution is easier to solve. However, conceptually the two models are observationally equivalent using data on employment, unemployment and the observed dynamic transitions among these states and the observed accepted wages.[24]

4.3. Structural estimates of on-the-job search models

Flinn (2002a) structurally estimated the above on-the-job search model to evaluate the welfare distributions in Italy and the US. Flinn (2002a) used an Italian and US (NLSY79) panel data of labor market histories ("working cycles") for a sample of men to estimate, via maximum likelihood, the parameters of the model. The results show that the job arrival rates, λ_u and λ_e, the job destruction rate, δ, and the standard deviation of offered wages are higher in the US than in Italy. Hence, the estimated search model is consistent with the observed fact that earnings are more dispersed in the US than in Italy—both due to higher variability in the wage offer distribution and the higher mobility (flexibility) in the US labor market. The most striking result of Flinn (2002a) is the model prediction that the present value lifetime inequality is much lower than the earnings inequality and, moreover, that the lifetime earnings inequality in the US is *lower* than in Italy.[25] There is no other structurally estimated economic model that could test directly the well-known point made by Friedman (1962) that we have to distinguish between short and long run income inequality when we compare economies with different labor market mobility. Flinn (2002a) is the first attempt to measure the potential sources and welfare implications

---

[24]The finite horizon dynamic programming labor supply methods and applications are summarized in Eckstein and Wolpin (1989a), Eckstein and Wolpin (1989c), Keane and Wolpin (1994) and Wolpin (1995).

[25]The expected present value of life time welfare is measured as follows. For each labor market environment one million histories were randomly created. For each history the initial present discounted utility of being unemployed is calculated, looking forward for 45 years. One can contrast the distribution of this welfare measure to the distribution of wages in the cross-sectional steady state for each economy.

related to the vast empirical literature on earnings inequality comparisons over time and countries (Katz and Autor, 1999).[26]

Wolpin (1992) extends the search model of Wolpin (1987) to include the option of on-the-job-search in order to study the importance of work experience on wage offers for blacks and whites during the first five years after graduating from high school. The model is nonstationary due to the finite horizon and the assumption that the mean of the wage offer distribution and the on-the-job offer probabilities depend on experience and other endogenous work history variables. Wolpin (1992) uses quarterly data for the five years post schooling and the optimization over 40 years. Work history per individual is set by working cycles (see above description) and the parameters are estimated using the likelihood of the joint observation of the work history and wages for all high school graduate males in the panel (NLSY79), who did not go to college. Wolpin distinguishes between employer specific experience and general experience and finds that the sum of the two is higher for whites than for blacks. However, the blacks have higher offer rates of jobs while unemployed and employed. Since observed wages of blacks increase less than those of whites, work experience affects the lifetime earnings profile of people more than the job offer arrivals in employment.

5. Search, matching and bargaining

5.1. Models and empirical implementation

An important extension of the simple search model above concerns the models where all workers and firms are ex ante the same, but when a worker meets a firm they sample a value of the *match* between them (Diamond and Maskin, 1979; Jovanovic, 1979). The value of this *match* is a random variable that represents the worker productivity, p_{ij}, such that $x_i = z_j = 1$, and it is sampled randomly from a given distribution function, $H(p)$, that is independent of worker and firm i and j. In addition, it is usually assumed that there is no search on the job, so $\lambda_e = 0$. Here we follow the general framework and assume that V^{lu} and V^{fu} are the expected present value of the worker search for a job and the firm vacancy to search for the worker where both have the same discount factor, ρ.

This extension leads towards an equilibrium specification that is more sophisticated than the Lucas and Prescott (1974) analysis of the search model. The conceptual equilibrium issue is how the match value is divided by the worker and the firm. In Diamond and Maskin (1979) and Mortensen (1982) it is assumed that firms and workers search for each other in a symmetric way, that is, firms solve a search problem that is equivalent to that solved by the unemployed worker. Wolinsky (1987) provides a very clear analysis of this model solved for equilibrium using the cooperative Nash axiomatic framework and the strategic Rubinstein's bargaining model. Here, as in Eckstein and Wolpin (1995), it is assumed that the value of the match is divided by a Nash axiomatic bargaining equilibrium solution.[27] The Nash axiomatic bargaining solution is characterized by a reservation

---

[26]Bloemen (2005) estimates a model with on-the-job-search and endogenous search efforts using simulated maximum likelihood. His focus is on the effect of variation in exogenous variables that affect the cost of search.

[27]In Wolinsky (1987) and Eckstein and Wolpin (1995) this model is solved with endogenous search efforts, s, by workers and firms. This extension of the general framework goes back to work well-covered by Mortensen (1986). Endogenous search efforts by firms and workers is theoretically simple to add but not always simple to solve, but it contains several interesting empirical implications that we discuss below.

match-productivity level p^* and wage and profit functions that satisfy

$$p^* = \rho V^{\text{lu}} + \rho V^{\text{fu}},$$

$$w(p) = \rho V^{\text{lu}} + \alpha(p - p^*),$$

$$\pi(p) = \rho V^{\text{fu}} + (1 - \alpha)(p - p^*), \tag{15}$$

where V^{ju} is the expected present value of search of party j and $\alpha \in (0, 1)$ is the bargaining power of workers. The reservation wage is now given by $w^* = w(p^*) = \rho V^{\text{lu}} + \alpha(p^* - p^*) = \rho V^{\text{lu}}$, which is exactly the same as in the simple model. The actual wage (or profit) per worker is equal to the reservation wage (or the reservation profit) plus the bargaining power (α or $(1 - \alpha)$) of the productivity surplus of the match, $(p - p^*)$.

Following Eckstein and Wolpin (1995) it is clear that the model is observationally equivalent to the search model, if one uses the same data on durations of unemployment and accepted wages, without additional data on the firms and the market. That is, here the structural productivity is given by distribution $H(p)$ rather than the c.d.f. of w, say $F(w)$, as in the simple search model. Hence, one has to use Eq. (15) to find $F(w)$ from the given distribution for p. The only difference is that Eq. (15) implies that the wage is a nonproportional function of productivity, and, hence, the exact equivalence to the simple search model is achieved only at a very special case where the firm and the worker have exactly the same optimization problem. In this case, $\alpha = 0.5$ and the parameters related to the firm and the worker value functions are the same. That is, if the productivity of the match p is divided by a constant fraction (say, one half) then the model is equivalent to the simple search model described above (see also Flinn and Heckman, 1982). Hence, we can interpret the simple search model as an equilibrium matching model where the wage rate w is a fixed fraction of the worker-firm random productivity (match) value.

Clearly, the special case where $\alpha = 0.5$ and the parameters related to the firm and the worker value functions are the same is not the standard economic case. In the general case, the likelihood function conditional on a sample of durations of unemployment and accepted wages for I workers, $\{\tau_i, w_i, i = 1, \ldots, I\}$, is the product of $\Pr(\tau_i, w_i)$. There are two identification problems. First, by analogy to the non-recoverability problem in the classic job search model (see Section 3), the left-hand tail of the productivity distribution is not nonparametrically identified. As we shall see in Section 6, this problem carries over to other equilibrium search models with heterogeneous productivities. Secondly, the bargaining parameter is not identified without additional structure (Eckstein and Wolpin, 1995; Flinn, 2002b).

With an identified specification, the calculation of $\Pr(\tau_i, w_i)$ for evaluating the likelihood function can be done by numerical integration or using the method of simulated maximum likelihood (McFadden, 1989), by analogy to Section 3.1. If p is log-normal then $w(p)$ is not log-normal if $\rho V^{\text{lu}} + \alpha p^* \neq 0$, and one has to use numerical integration to get the likelihood function. Eckstein and Wolpin (1995) show that the estimates of the bargaining power (α) and the search cost function using only data on unemployment durations and accepted wages are not robust.

To sum, given data on durations of unemployment and accepted wages of the first job, the search-matching-bargaining model has the same empirical implications (concerning identification and potential fit to the data) as the standard simple search model above.

5.1.1. The Pissarides model

The basic Pissarides (1990) model considers a labor market with a fixed continuum of workers. The (endogenous) measures of unemployed workers and vacancies are denoted by U and V. With fixed search intensities, the flow of contacts is given by the matching function $m(U, V)$, so that the job offer arrival rate λ equals $m(U, V)/U$. From the point of view of an employer with a vacancy, the arrival rate of workers equals $\lambda U/V$.

As in the previous model, in a given equilibrium, U and V are constant, so that the matching function has no empirical significance, unless one compares different equilibria. However, in case of policy changes, U and V may change. It turns out that policy effects and other equilibrium implications depend strongly on whether the matching function displays constant returns to scale or not. This has generated a vast literature in which matching functions are estimated. The matching function is identified by exploiting variation in U and V across different labor market equilibria.[28]

This baseline Pissarides model differs in two ways from the models considered earlier in the present Section 5.1. First, there is no match-specific heterogeneity. Secondly, the contact arrival rates are functions of the number of searching agents at both sides of the market, by way of a matching function, and the number of vacancies is endogenized by way of a free entry condition for firms. Due to the first aspect, there is only one equilibrium wage outcome. Clearly, this model is more amenable to macro-economic calibration analyses focusing on equilibrium effects of policy changes than to micro-econometric analyses focusing on heterogeneous outcomes at the individual level. However, whether we allow for match-specific heterogeneity (as described above) or not, the model is used in structural empirical analysis belongs to the same general framework of Section 2.

5.2. Structural estimates of search-matching-bargaining models

Eckstein and Wolpin (1995) empirically estimate the return to schooling using data on the post-schooling duration to the first job and the accepted wage for that job. Schooling is assumed to be exogenous and divided into four levels: high school dropouts; high school graduates; some college, and college graduates. The return to schooling is defined in terms of the differences in mean *offered* wages between consecutive schooling levels, because the mean offered wage is an accurate indicator of the market value of productivity. The main point is that when labor supply is governed by the search-matching-bargaining model, the differences in *observed accepted* wages are biased estimates for the return to schooling. It is assumed that the model is symmetric for workers and firms and allows for unobserved heterogeneity of five types (i.e. a discrete distribution with five points of support) for each schooling level. The main result is that the observed mean differences in accepted wages substantially underestimate the return to schooling using the NLSY79 data for blacks and whites separately. It should be emphasized that theoretically the bias could be either way.

Eckstein and Wolpin (1999) discuss the estimation strategy of the search-matching-bargaining model that interprets the bargaining power parameter as a source of labor market discrimination between blacks' and whites' gaps in wages and unemployment. Their results indicate that the bargaining power parameter for whites is between 40% and 56% higher than that for blacks. As a result, the discrimination that is due to the difference

---

[28]Petrongolo and Pissarides (2001) survey the estimates of the matching function $m(U, V)$.

in bargaining power can account to the entire wage gap (as an upper bound measure) between blacks and whites with the same schooling level.

Flinn (2002b) uses the search-matching-bargaining model to show how the minimum wage affects the entire accepted wage distribution by affecting the bargaining position of firms and workers. He uses CPS data to assess the welfare impact of the 1997 change in the US minimum wage but does not fully use the observed durations and accepted wages to estimate the model with maximum likelihood.

Recently, some papers have developed and estimated models that allow for wage bargaining as well as on-the-job search (Postel-Vinay and Robin, 2002; Cahuc et al., 2004; Dey and Flinn, 2005). Some of these studies identify and estimate the bargaining power parameter. We defer a discussion to the next section.

6. Equilibrium search with wage posting

6.1. Background

Recently, a literature has emerged in which equilibrium search models are estimated (see van den Berg, 1999, for a survey). In equilibrium search models, the wage offer distribution is endogenous. It results from optimal wage setting by firms that take account of the behavior of job seekers and other firms. The most important difference between these models and the search-matching-bargaining model and the simple search model above is that here a firm posts a wage prior to meeting a potential worker. The parameters of the endogenous wage distribution are fully determined by the productivity, search friction and preference parameters. In Section 5, a firm and a worker determine the wage using a bargaining procedure over the surplus. In the Lucas and Prescott (1974) simple search model the wage dispersion is fully determined by the heterogeneity of productivity across firms and the workers' human capital, but there is no non-trivial effect of the amount of search frictions on the wage offer distribution. In all cases, the productivity distribution is determined exogenously, the worker job acceptance strategy determines the lower observed wage for each "type" of worker, and the equilibrium determines the truncation and location of the wage distribution relative to the productivity distribution.

Diamond (1971) made the point that if homogeneous firms (retailers) and workers (consumers) maximize profit and lifetime income (utility), respectively, then the equilibrium (perfect information) offered wage (price) by firms (retailers) is the unique reservation wage (price) of workers (consumers). This claim generated much criticism on the simple search model, which led to the posting equilibrium search analysis. It turned out that under limited information the equilibrium wage in the search model is consistent with wage dispersion. In general, equilibrium wage offers are equal to the reservation wage of some (group of) worker(s). Thus, a model in which potential workers at a firm differ in their reservation wage values may generate wage dispersion. The underlying reason for the latter is that there is a trade-off between the profits per worker and the steady-state number of workers at the firm. In equilibrium, some firms may choose to set a high wage (giving a large workforce but small profits per worker) while others may prefer to set a low wage (giving a small workforce but high profits per worker). In general, the equilibrium wage (offer) distribution depends on all the parameters of the model including the search frictions in the form of the job arrival rates (λ_u and λ_e) and the distributions of worker and firm heterogeneity.

Basically, two approaches leading to equilibrium wage dispersion can be distinguished in the theoretical literature, depending on the source of the reservation wage heterogeneity. In the first approach (Albrecht and Axell, 1984; Eckstein and Wolpin, 1990), workers are heterogeneous in their opportunity cost of employment b. This implies heterogeneity of the unemployed workers' reservation wages using the same reservation wage strategy as we described above. In the second approach (Mortensen, 1990; Burdett and Mortensen, 1998), ex ante identical workers are allowed to search on the job. As we will explain below, this generates ex post heterogeneity of reservation wages across currently employed workers.

The equilibrium search models provide a framework to empirically analyze the sources of wage dispersion: (a) workers heterogeneity (observed and unobserved); (b) firm productivity heterogeneity (observed and unobserved); (c) market frictions. The equilibrium framework can combine together the simple search on-the-job model, the search-matching-bargaining and the posting equilibrium models to empirically measure the quantitative importance of each source for the observed earnings and welfare dispersion as well as employment, unemployment and labor market mobility. The papers that we describe below illustrate what has been done and what could be learned further from this approach.

6.2. Models

For expositional reasons we start with the basic Burdett and Mortensen (1998) model, even though the theoretical and first empirical analyses of the Albrecht and Axell (1984) model predate the analyses with this model. The model considers a labor market consisting of fixed continuum of homogeneous workers (so, in the framework of Section 2, $I = 1$) and homogeneous firms. The measure of workers is denoted by m, and the endogenous measure of unemployed workers by u. The measure of firms is normalized to one. The supply side of the model is equivalent to the classic search model with on-the-job search. As a result, the workers' optimal strategy is as in Section 4.1, and the unemployed workers' reservation wage is given by Eq. (14).

The construction of the equilibrium model starts with the equilibrium flows of workers for the wage posting model. It should be noted that these conditions could be imposed on all the models above and provide additional restrictions that could be used for identification of model quantities. Next, we describe the firms' strategy and construct the equilibrium.

6.2.1. Equilibrium flows of workers

Firms do not offer a wage below w^*, so that all wage offers will be acceptable for the unemployed.[29] Consequently, the flow from unemployment to employment is $\lambda_u u$. The flow from employment to unemployment is $\delta(m - u)$. In a steady state, these flows are equal and the resulting rate u/m of unemployed workers equals

$$\frac{u}{m} = \frac{\delta}{\delta + \lambda_u}. \tag{16}$$

---

[29]This simple point made by Eckstein and Wolpin (1990) is relevant for the literature on nonparametric identification of the wage offer distribution where certain wages are not observed due to endogenous selection of jobs by workers. In the homogeneous equilibrium, wages that no one will accept will not be offered.

Let G be the "accepted earnings" distribution of wages paid to a cross-section of employees. These wages are on average higher than the wages offered, because of the flow of employees to higher paying jobs. The stock of employees with a wage less than or equal to w has measure $G(w)(m - u)$. The flow into this stock of unemployed who accept a wage less than or equal to w, and this flow is equal to $\lambda_u F(w) u$. The flow out of this stock consists of those who become unemployed, $\delta G(w)(m - u)$, and those who receive a job offer that exceeds w, $\lambda_e(1 - F(w))G(w)(m - u)$. In the steady state, the flows into and out of the stock are equal, so

$$G(w) = \frac{\delta F(w)}{\delta + \lambda_e(1 - F(w))}, \tag{17}$$

where we have substituted for u using Eq. (16). Eqs. (16) and (17) are equilibrium flow conditions.

6.2.2. Firms' behavior

Firms post wage offers and they do not bargain over the wage. We assume that the wage and the marginal value product p do not depend on the identity of the employees or the number of employees. In the context of the framework of Section 2, p is a fixed constant model parameter while w and the steady-state labor force of a firm l only depend on an index j denoting the firm. For convenience we omit the index j. A firm chooses w by maximizing the steady-state profit flow $(p - w)l(w)$ over w. In equilibrium, the firm sets w below p because it can always attain a positive profit by setting $w = w^*$. At a wage $w < p$ it is profitable to expand, so all firms always have a vacancy. As a result, the distribution of wages across firms equals the wage offer distribution.

6.2.3. Equilibrium

In equilibrium two conditions are met. First, the labor force of firms at each wage w should be equal to the effective labor supply at w, such that,

$$l(w) \, dF(w) = (m - u) \, dG(w), \tag{18}$$

which, by using (16) and (17),[30] gives

$$l(w) = \frac{m\delta\lambda_u(\delta + \lambda_e)}{\delta + \lambda_u} \frac{1}{(\delta + \lambda_e(1 - F(w)))^2}. \tag{19}$$

Second, since all firms are equal, the equilibrium steady-state profit flow must be equal for all equilibrium wages. Since the lowest wage in the market must be w^*, this condition amounts to

$$(p - w)l(w) = (p - w^*)l(w^*), \tag{20}$$

for all w in the support of F, which can be solved for F given w^*,

$$F(w) = \frac{\delta + \lambda_e}{\lambda_e}\left(1 - \sqrt{\frac{p - w}{p - w^*}}\right) \tag{21}$$

Eqs. (21) and (14) for F and w^* constitute the Bayesian–Nashfs non-cooperative steady-state equilibrium. Firms always offer wages that are smaller than their productivity level,

---

[30]See Burdett and Mortensen (1998) for technicalities like the absence of mass points in F.

so they have a certain monopsony power. Because of wage dispersion, workers make job-to-job transitions, each firm faces prospective workers who are heterogeneous in their current minimum acceptable wage, and this in turn rationalizes the mixed strategy equilibrium that leads to wage dispersion. Since all workers and firms are identical, the presence of wage dispersion implies that the law of one price does not hold in equilibrium. The wage dispersion is fully due to the presence of search frictions with limited information.

6.2.4. Wage posting and bargaining equilibria

To consider the similarities of the above equilibrium to the wage determination in the bargaining models of the previous section, we define the random variable y as

$$y = \frac{p - w}{p - w^*}, \tag{22}$$

with w the wage in a cross-section of workers. It follows from the above equations that the density of y is

$$g_y(y) = \frac{\psi}{2(1 - \psi)} y^{-3/2} \quad \text{for } \psi^2 \leqslant y \leqslant 1 \tag{23}$$

with $\psi = \delta/(\delta + \lambda_e)$. Here, $y = 1$ corresponds to the bottom and $y = \psi^2$ to the top of the job ladder. The distribution of the random position on the ladder y only depends on λ_e/δ, the expected number of job offers during an employment spell, and this is clearly a measure of the speed at which employees climb the ladder. If we rewrite Eq. (22) as

$$w - w^* = (1 - y)(p - w^*), \tag{24}$$

we see that in equilibrium the value of the match $p - w^*$ is split between the worker and the firm with $1 - y$ being the worker share, similar to Eq. (15). The latter is a random variable with a distribution that depends on the speed at which alternative offers are obtained. This analysis[31] clearly shows the similarity between wage determination in the wage posting equilibrium search model and wage determination in the bargaining models of the previous section: the surplus of the match is divided in accordance to the relative market power of the agents.

Note also that the mean of the relative mark-up y satisfies $E(y) = \psi = 1/(1 + \lambda_e/\delta)$. Hence, it follows that ψ or/and λ_e/δ are sensible measures of the labor market frictions, as they capture the extent to which firms are able to exploit such frictions when they set wages.[32]

The two main empirical shortcomings of the model are that it predicts an increasing density of the cross-sectional wage distribution G and that it predicts a constant exit rate out of unemployment h (since $h = \lambda_u$). These implications are not consistent with almost all data sets. This has led to a series of papers that make both theoretical and empirical contributions by way of developing and estimating extensions of the basic model. To overcome the first shortcoming, the authors allowed firms in the model to be heterogeneous with respect to p. In general, the equilibrium solution for a market with heterogeneous agents differs from the solution for the homogeneous model. Bontemps

---

[31]Borrowed from van den Berg and Ridder (1998).

[32]Ridder and van den Berg (2003) work this out in a cross-country comparative empirical analysis. Mortensen (2003) and van den Berg and van Vuuren (2003) also use λ_e/δ to quantify search frictions.

et al. (2000) provide a comprehensive analysis of the model with a general continuous distribution for p. Interestingly, the set of G that can be generated by varying the productivity distribution in the model over all possible continuous distributions can be characterized by the restriction that the wage density does not increase as fast as in the basic Burdett and Mortensen (1998) model. This is obviously good news from an empirical point of view.

The second shortcoming of the basic Burdett and Mortensen (1998) model (a constant exit rate out of unemployment) can be dealt with by allowing for heterogeneity across individuals of determinants of the exit rate (see Section 3). An example for this is the heterogeneity in the value of leisure, b, as in Albrecht and Axell (1984). Their model imposes that $\lambda_e = 0$ and that individuals are heterogeneous by nature with respect to their value of b. It should be stressed that b is assumed to be a time-invariant individual-specific characteristic. This excludes dependence of the individual's b on past wages. As we have seen, heterogeneity in b may generate equilibrium wage dispersion. Each point of support of F equals a reservation wage of an unemployed worker type. The distributions F and G are again different from each other, but now this is not because of job-to-job transitions but because unemployed individuals with high b only flow to jobs with high wages.

Clearly, the Albrecht and Axell (1984) model does not allow for job-to-job transitions. However, contrary to the basic Burdett and Mortensen (1998) model, the model allows for equilibria in which at least some unemployed workers reject at least some of their job offers some of the time, because some firms may offer a wage smaller than the highest levels of b in the market. Moreover, changes in the unemployment benefit levels may affect the unemployment duration distribution. Due to the heterogeneity in the unemployed workers' values of b, the aggregate unemployment duration distribution displays negative duration dependence.

The above models have been merged and extended. Mortensen (1990) extends the basic Burdett and Mortensen (1998) model by allowing both b and p to be heterogeneous. Postel-Vinay and Robin (2002) assume workers as well as firms to be heterogeneous in productivity and also allow for on-the-job search. Firms and workers bargain over the wage. Firms hold all the bargaining power, but when an employed individual receives an outside offer, the two firms enter a Bertrand competition game which is won by the most productive firm. In some ways this model is richer than the previous models: it allows for endogenous wage growth on the job and also for job-to-job transitions that can result in a wage cut. It also leads identical workers to earn different wages at the same firm. The authors show that under certain assumptions the equilibrium accepted wage distribution has the well-known Mincerian functional form, that is, $\ln w = \ln(\varepsilon) + (search\ friction, p)$, where ε is the index of worker's productivity and p is the firm productivity.

Clearly, wage determination in Postel-Vinay and Robin (2002) shares some features with the bargaining models discussed in Section 5.1. However, in Postel-Vinay and Robin (2002) the workers' bargaining power is essentially zero, and yet wages may exceed the unemployed reservation wage due to the competitive pressure of firms poaching employed workers.

6.3. Structural estimation

6.3.1. The basic Burdett and Mortensen (1998) model

van den Berg and Ridder (1993b, 1998) and Kiefer and Neumann (1993) estimated the basic model before proceeding towards more complicated models. The model has four

unknown parameters: $\lambda_u, \lambda_e, \delta$ and p. As in the simple search model, if b and/or ρ are unobserved then w^* constitutes a fifth unknown parameter. Observable durations, wages and exit destinations are all endogenous according to the model, and their distributions depend on the model parameters. Similar to the search on-the-job model, the wage data, if the job spells concern the first job after a spell of unemployment, then the corresponding wages are random drawings from the wage offer distribution F. If the job spells concern jobs occupied by a random sample of all workers who are employed at a certain point in time, then the wages are random drawings from G.

Consider a random sample of labor force participants, containing, for unemployed individuals, the remaining unemployment duration, and, for employed individuals, the current wage, the remaining job duration and the labor market state occupied after the current job. Mortensen (1990), Kiefer and Neumann (1993) and van den Berg and Ridder (1993a) show that these data identify the parameters. Identification can be illustrated by examining an estimation method that links the observables as directly as possible to the model primitives. For this, it is instructive to first derive the likelihood function. Note that in the above sample, the unemployment duration has an exponential distribution with parameter λ_u, the cross-sectional wage is a random drawing from G (we denote its density by g), the job duration given the wage w has an exponential distribution with parameter $\delta + \lambda_e(1 - F(w))$, exit to unemployment given job exit has probability $\delta/(\delta + \lambda_e(1 - F(w)))$, and whether one is unemployed or not has a Bernoulli distribution with parameter $\delta/(\delta + \lambda_u)$. A likelihood contribution then equals

$$\frac{1}{\lambda_u + \delta} (\delta \lambda_u e^{-\lambda_u \tau})^{d_1} (\lambda_u g(w) \delta^{d_2} (\lambda_e(1 - F(w)))^{1-d_2} e^{-(\delta + \lambda_e(1-F(w)))\tau})^{(1-d_1)}, \tag{25}$$

where $d_1 = 1$ ($d_1 = 0$) if the individual is initially unemployed (employed), and $d_2 = 1$ ($d_2 = 0$) if an initially employed individual moves to unemployment (to another job) after job exit. We use τ as a general symbol for a duration variable. The likelihood function is then the product of the contributions over individuals, where we substitute the model expressions for F and g. It is instructive to relate this to the likelihood functions in Sections 3 and 4. In the setting of the basic model of Section 3 we are only concerned with individuals with $d_1 = 1$. For these we also observe the accepted wage, which is here a random drawing from F, and so the likelihood contributions need to be augmented with the density $f(w)$ of F. In the setting of a basic model of Section 4 we are only concerned with individuals with $d_1 = 0$. In the case $d_2 = 0$ one typically also observes the accepted wage upon a job-to-job transition, which is a random drawing from F truncated from below at w, so the likelihood contributions need to be augmented with $f(\widetilde{w})/\overline{F}(w)$, where $\widetilde{w}$ is the accepted wage. Note that the support of F and G depends on the unknown parameters.

Now that we have derived the likelihood function, we can return to the discussion on identification. Suppose one would estimate the model in two steps. In the first step, F is treated as an unknown distribution to be estimated nonparametrically (e.g. by way of kernel estimation) along with the event arrival rate parameters, so we substitute g as a function of F and the arrival rate parameters in the above expression. Further, w^* is estimated as the lowest observed wage. Clearly, the parameter λ_u is then identified from the unemployment durations, and δ and λ_e are identified from the job durations ending in transitions to unemployment and to another job, respectively. In the second step, p is then identified by equating e.g. the mean of the theoretical F to the mean of the estimated F.

In fact, the model is heavily overidentified with these data. For example, information on F is contained both in the wage sample and in the way in which the elasticity of the job-to-job transition rate with respect to the wage varies with the wage. Moreover, p is also identified from other moments of F. This reflects the "cross-equation restrictions" in the model: the parameters affect both the wage distributions and the duration distributions. In practice, this information is used to identify the heterogeneity distributions in richer model specifications (see below).

The unknown parameters can be estimated simultaneously with maximum likelihood (ML)-type procedures. The fact that the support of F and G depends on the unknown parameters means that ML estimators have non-standard properties, like in the estimation of the classic search model of Section 3. The resulting estimates are obviously sensitive to outliers (i.e. measurement errors) in the wage data. One may deal with this by allowing for wage measurement errors in the empirical model specification (van den Berg and Ridder, 1998). Also, some job-to-job transitions result in a job with a lower wage, and (as in Section 4) one may also handle this by allowing for measurement errors in the wage data. Alternatively, one may include into the model an exogenous transition rate to jobs with a lower wage (Ridder and van den Berg, 1997), or one may include non-wage job characteristics into the model. The Postel-Vinay and Robin (2002) model can generate wage cuts because workers do not only care about the wage but also about the firm-specific productivity.

The results in van den Berg and Ridder (1993b) show that robust estimates can only be obtained if duration and wage data are jointly used.[33] Even so, we already know from Section 6.2 that not too much should be expected from the fit of this model to labor market data. We now proceed by examining the estimation of generalizations of the basic Burdett and Mortensen (1998) model.

6.4. Estimation of models with heterogeneity; data on workers

Eckstein and Wolpin (1990) estimate the Albrecht and Axell (1984) model with heterogeneity, both of firms' p and of workers' b. This was the first structural empirical analysis of an equilibrium search model. The first goal of Eckstein and Wolpin (1990) was to demonstrate the feasibility of estimating a labor market equilibrium model using only data on workers. The second goal was to use the model to interpret the joint wage and duration distributions for workers who are supposed to be homogeneous in their market productivity. The likelihood function, using the duration and the accepted wage data, has a similar form to that of the simple search model (9) but with a discrete unobserved heterogeneity distribution for b . As noted, the model is rich in terms of explaining unemployment durations, and the estimated version fits the duration data well. However, the estimated unobserved heterogeneity of workers from the duration data had difficulty explaining the shape of the observed accepted wage distribution (Eckstein and Wolpin, 1990). This is due to the fact that each point of support of the wage offer distribution necessarily equals a reservation wage of an unemployed worker type. Allowing for heterogeneity in p across firms did not help here, and the measurement error accounted for almost all of the dispersion in observed wages.

---

[33]This is also true for more general models; in fact, those may not even be identified from either data separately.

van den Berg and Ridder (1998) estimated an extended version of the Burdett and Mortensen (1998) model. They assume that the labor market is segmented and consists of a large number of separate different submarkets within which workers and employers are homogeneous. This means that observed market outcomes are statistical mixtures of market outcomes in the basic Burdett and Mortensen (1998) model. To distinguish the heterogeneity in the van den Berg and Ridder (1998) model from heterogeneity within a given market, one may call the former "between-market" heterogeneity. It does not imply that a firm can only use one specific type of labor, provided that the firms' production functions are additive in their different types of labor inputs. In this case, p is a worker characteristic rather than a firm characteristic. This contrasts models with productivity heterogeneity "between markets" to models with productivity heterogeneity "within the market", as in the latter case the productivity is a firm characteristic. In practice, segments can be defined by observed individual characteristics x like profession and level of education (e.g. each combination of profession and level of education defines a segment) as well as by unobserved characteristics. Allowing for between-market heterogeneity in p enriches the model by allowing for structural unemployment of workers whose p falls short of the wage floor (i.e., unemployment benefits or a mandatory minimum wage).

Suppose that segments are at least partly defined by unobserved characteristics. The distributions of the structural parameters across workers in different segments are then estimated along with any other model parameters. The likelihood is obtained by integrating ("mixing") the likelihood associated with a homogeneous model with respect to the distributions of the parameters. Note that in this case the distributions of wage offers and wages among employed workers are mixtures of the corresponding distributions in the basic model. As may be expected, mixtures over the productivity parameter p provide an accurate fit to the wage data. The model also predicts negative duration dependence of the observed exit rate out of unemployment. Some individuals have a positive exit rate λ_u whereas the structurally unemployed have a zero exit rate, and this unobserved heterogeneity in the individual exit rate causes the observed exit rate to decrease as a function of duration.

Note that the distribution of p is not fully identified, as we only observe economic activity in segments where p exceeds the wage floor. This is a deep version of the non-recoverability problem in estimation of the classic job search model (see Section 3). Instead of the left-hand tail of the wage distribution it is now the left-hand tail of the productivity distribution that is not identified. It implies that the effect of a small increase in the wage floor is identified but the effect of a small decrease is not, unless one is prepared to make functional-form assumptions.[34] This insight carries over to other equilibrium search models with heterogeneous productivities. Predictions on outcomes due to counterfactual policies that reduce the reservation wage have to be based on functional forms. But this is not different to any other case where the counterfactual concerns outcomes that have never been observed.

---

[34]However, the *amount* of probability mass below the wage floor is now identified from the unemployment duration data, since it corresponds to the fraction of permanently unemployed individuals (this is exploited by e.g. Ridder and van den Berg, 2003). This result neatly illustrates the interrelations between wage and duration variables in equilibrium search models, as well as the potential they offer for fruitful exploitation in empirical inference.

The results indicate that job arrival rates are important determinants of the unemployment duration and the unemployment level and that job arrival rates for unemployed is only slightly higher than that for employed workers (about 0.04 per week).[35] The estimates are used to decompose wage variation into variation due to search frictions (i.e., the variation present in the homogeneous model) and the additional variation due to heterogeneity across segments. Typically, the results indicate that at least 50% of wage variation is due to variation in productivity across segments, and at most 25% is due to search frictions. Hence, the main source for wage variation is due to productivity heterogeneity as it is in the simple search model and not to the non-competitive structure of the posting equilibrium structure of the model. Furthermore, they find that on average the wage mark-up is about 13% the competitive wage, and a policy experiment of increasing the minimum wage by 25% implies a 16% increase in unemployed workers.

6.5. Estimation of models with heterogeneity: data on workers and firms

Bontemps et al. (2000) develop a model with a continuous distribution of within-market heterogeneity of p, and they develop and apply an estimation method that provides a nonparametric estimate of the productivity distribution.[36] In the first step, the "transition" parameters λ_u, λ_e and δ are estimated along the lines of the identification argument in the above subsection.[37] In the second step, the productivity distribution is estimated from the nonparametric wage data distribution, using the relationship between p and w that follows from the firms' wage posting behavior in the model. Note that the estimates from the first step are only based on those parts of the model that describe worker behavior. The second step then exploits the first-order condition of the firms to estimate the productivity distribution. The estimates of the transition parameters can thus be expected to be consistent under a wide range of models of firm behavior and wage determination. The estimation method provides a *perfect* fit to the wage data (if the model is correct). This estimation method has since been used by a number of other studies (see e.g. Mortensen, 2003) and serves as a starting point for the development of estimation methods for more complicated equilibrium search models.

Using the worker data, Bontemps et al. (2000) find that the monthly arrival rates for the unemployed are about 0.07 and that the firms' wage mark-ups are large on average, in particular for the firms at the right tail of the distribution. Firm data are used to assess the qualitative and quantitative implications of the model and the estimates. For example, the firm data support the implication of the model that the wage and the profit share are increasing with productivity.[38]

---

[35]These estimates are lower than the Blau (1989) estimates for the US. These findings are consistent with Flinn (2002a) and the wide claim that the US labor market has more labor mobility than in Europe.

[36]One motivation for their work stems from their observation that if simple functional forms like a Pareto or log-normal distribution are adopted for p then this often does not give a good fit to the whole wage density. This is true even within segments based on occupation or industry. A Pareto distribution for p gives a better fit than (log-)normal distributions or other popular distributions with a few parameters.

[37]In fact, G is estimated nonparametrically from cross-sectional wage data, and this is used as input in the estimation of the transition parameters. Standard errors are obtained by bootstrapping.

[38]By now, dozens of studies have appeared in which similar models are estimated. Bontemps et al. (1999) estimate a model where both b and p have a continuous distribution. Mortensen (2003) is a detailed empirical study of the Burdett and Mortensen (1998) and Bontemps et al. (2000) models, using Danish matched employee–employer panel data on flows and wages.

Postel-Vinay and Robin (2002) estimate their model using a French matched employer–employee panel data set on productivity, flows and wages. They use the estimated model to decompose the wage variance between the three potential sources: workers' personal heterogeneity, productivity heterogeneity, and market search frictions. A striking result is that workers' skill differences account for 50% of the wage variance in high-skill jobs, but for 0% in low-skill jobs. The market imperfection impact on the wage dispersion is typically 50% for all skills.

It seems obvious that the more complicated the equilibrium model, the more information is needed to estimate the size of the gap between a wage and its underlying productivity, but the literature shows that this view is an over-simplification. The Lucas and Prescott (1974) model, which is one of the starting points of the literature, equates wage and productivity, so that the gap is zero by assumption. Search-matching-bargaining models are more realistic in that they allow for a gap between wage and productivity. Specifically, they predict that the wage is some fraction of the productivity. However, we have seen that the bargaining parameter that determines this fraction is hard to identify from worker data only. The Burdett and Mortensen (1998) model and other wage posting models based on it predict that the gap between wage and productivity is driven by the amount of frictions in the market, and the latter is identified from worker data. The Postel-Vinay and Robin (2002) model distinguishes between different productivity components, and both worker and firm data are required.

Recall that wage determination in Postel-Vinay and Robin (2002) shares some features with search-matching-bargaining models. The workers' bargaining power is essentially zero, but wages exceed the unemployed reservation wage due to the competitive pressure of firms poaching employed workers. This raises the question as to whether match-bargaining models with on-the-job search can be constructed and identified in which employers enjoy *less* than all of the bargaining power and still must compete for workers with poaching firms. The recent studies by Dey and Flinn (2005) and Cahuc et al. (2004) address this and demonstrate that indeed it is possible to identify the bargaining power parameter.

7. Concluding comments

The empirical labor search models that are described in this survey provide a dynamic stochastic framework for the empirical analysis of most questions that labor economists consider. Furthermore, one can nowadays use cross section, worker panel, firm panel and matched employee–employer data to analyze equilibrium labor market models. These models can jointly consider labor supply issues related to unemployment, job mobility and wage dispersion as well as labor market discrimination and unemployment wage gaps. Furthermore, the models can be applied to the analysis of education policy and the return to education (see also the review by Wolpin, 2003).

Is there any empirical evidence in favor of the posting/bargaining equilibrium search models vs. the simple Lucas and Prescott equilibrium interpretation? Until recently, the answer was: not really. The empirical research based on longitudinal labor supply data shows that productivity heterogeneity among firms is a useful assumption to fit the data. From such data, partial job search models are just identified under a functional form assumption on the wage offer distribution. The Lucas and Prescott (1974) model, where the wage simply equals the productivity, can then be seen as a useful simple benchmark equilibrium search model with minimal assumptions.

Of course, the simple Lucas and Prescott equilibrium interpretation ignores the fact that firms often have an incentive to pay below productivity precisely because of search frictions. It certainly matters for many policies (like minimum wage policies and other policies that directly intervene in price formation) precisely how the relation between wages and productivities is in reality. Moreover, the reasoning in favor of the Lucas and Prescott (1974) model is based on longitudinal labor supply data. Equilibrium search models provide a foundation for wage variation by way of a productivity distribution, so if the mapping between wages and productivities is identified then, by implication, longitudinal labor supply data also just identify the productivity distribution.[39] But firm or establishment level data and matched worker-firm data provide additional, over-identifying, information. Firm data may provide observations on productivities and wages. For example, van den Berg and van Vuuren (2003) combine Danish worker and firm data to examine the effect of frictions on the mean wage offered by firms in specific submarkets. It turns out that the effect is significantly negative. Wages are smaller than productivities, and the difference is larger if frictions are large. It should be pointed out that the quantitative effect of frictions on the sector-specific mean wage is small.

What have we learned about the sources of wage dispersion? All empirical studies conclude that most wage variation is due to productivity variation. However, it would be too early to conclude from this that frictions are irrelevant for wage dispersion. First, many models predict that it is the interaction of frictions and productivity variation that explains wage dispersion. Productivity heterogeneity by itself cannot explain wage dispersion in the Burdett and Mortensen (1998) type of models, since in the absence of on-the-job search the degenerate Diamond (1971) wage solution applies. Secondly, reduced-form decompositions of wage variation typically assume that job-to-job mobility is not driven by the search for opportunities that are created by frictions.

What have we learned from the analysis of search models concerning the dynamic selection bias in the estimated return to education? It seems that the OLS estimated coefficients of the Mincerian equation might be upward biased and that much of the ''return'' to investment in education is hidden in the ''black-box'' of the market frictions. That is, in labor markets for high skilled workers there is less friction due to higher arrival rates of jobs.[40] Interpreting the OLS correlations in the Mincerian earnings equation is a great challenge to labor economists. Dynamic optimization theory in general and search theory in particular provide a natural interpretation framework for this goal.

In addition to the above applications, partial and equilibrium job search models have been developed and estimated to study a whole range of topics where labor market imperfections and dynamic considerations are important. We mentioned discrimination, minimum wage policy, unemployment insurance, retirement and social security policy, and active labor market policies. Presumably, this is just the beginning of an expanding use of search models in the empirical labor literature.

Good data on many aspects of employment decisions are increasingly available. This opens up many avenues for research. One may think of the multi-dimensional nature of job offers, of simultaneous/directed search strategies for job offers, and of employment policies

---

[39]Recall, however, footnote 34.

[40]It should be noted that the literature that focuses on using instruments to correct for selectivity/endogeneity bias in estimating the return to human capital investment, using the Mincerian function, tends to ignore the labor market frictions that are discussed above (see Card, 2001).

like firing costs and employment subsidies. One may also think of empirical equilibrium search models that allow for wage-tenure profiles and/or for stochastic productivity growth. Also, the recent availability of the rich new data that match employees and employers requires a conceptual framework for the empirical analysis. At this point, equilibrium search theory is probably the only theory that empirical economists can use for this purpose. The survey here tells the researchers how far this literature has reached so far.

Acknowledgments

References

Albrecht, J.W., Axell, B., 1984. An equilibrium model of search unemployment. Journal of Political Economy 92, 824–840.

Berkovec, J., Stern, S., 1991. Job exit behavior of older men. Econometrica 59, 189–210.

Blau, D.M., 1991. Search for nonwage job characteristics: a test of the reservation wage hypothesis. Journal of Labor Economics 9, 186–205.

Blau, D.M., Robins, P.K., 1986. Job search, wage offers and unemployment insurance. Journal of Public Economics 29, 173–197.

Bloemen, H.G., 2005. Job search, search intensity and labor market transitions: an empirical analysis, Journal of Human Resources 40, 231–269.

Bontemps, C., Robin, J.M., van den Berg, G.J., 1999. An empirical equilibrium job search model with search on the job and heterogeneous workers and firms. International Economic Review 40, 1039–1074.

Bontemps, C., Robin, J.M., van den Berg, G.J., 2000. Equilibrium search with continuous productivity dispersion: theory and non-parametric estimation. International Economic Review 41, 305–358.

Burdett, K., 1978. Employee search and quits. American Economic Review 68, 212–220.

Burdett, K., Judd, K.L., 1983. Equilibrium price dispersion. Econometrica 51, 955–970.

Burdett, K., Mortensen, D.T., 1998. Wage differentials, employer size, and unemployment. International Economic Review 39, 257–273.

Cahuc, P., Postel-Vinay, F., Robin, J.M., 2004. Wage bargaining with on-the-job-search: theory and evidence. Working paper, University of Paris-I.

Card, D., 2001. Estimating the return to schooling: progress on some persistent econometric problems. Econometrica 69, 1127–1160.

Dey, M.S., Flinn, C.J., 2005. An equilibrium model of health insurance provision and wage determination. Econometrica 73, 571–627.

Diamond, P.A., 1971. A model of price adjustment. Journal of Economic Theory 3, 156–168.

Diamond, P.A., Maskin, E., 1979. An equilibrium analysis of search and breach of contracts, I: steady states. Bell Journal of Economics 10, 282–316.

Eckstein, Z., Wolpin, K.I., 1989a. The specification and estimation of dynamic stochastic discrete choice models. Journal of Human Resources 24, 562–598.

Eckstein, Z., Wolpin, K.I., 1989b. Dynamic labor force participation of married women and endogenous wage growth. Review of Economic Studies 56, 375–390.

Eckstein, Z., Wolpin, K.I., 1989c. On the estimation of labor force participation, job search and job matching models using panel data. In: Weiss, Y., Fisheilson, G. (Eds.), Advances of Theory and Measurement of Unemployment. Macmillan Press, London.

Eckstein, Z., Wolpin, K.I., 1990. Estimating a market equilibrium search model from panel data on individuals. Econometrica 58, 783–808.

Eckstein, Z., Wolpin, K.I., 1995. Duration to first job and return to schooling: estimates from a search-matching model. Review of Economic Studies 62, 263–286.

Eckstein, Z., Wolpin, K.I., 1999. Estimating the effect of labor market discrimination on first job wage offers. Review of Economics and Statistics 81, 384–392.

Engberg, J., 1991. The impact of unemployment benefits on job search: structural unobserved heterogeneity and spurious spikes. Working paper, Carnegie-Mellon University, Pittsburgh.

Ferrall, C., 1997. Unemployment insurance eligibility and the school to work transition in Canada and the United States. Journal of Business and Economics Statistics 15, 115–129.

Flinn, C.J., 2002a. Labor market structures and inequality: a comparison of Italy and the US. Review of Economic Studies 69, 611–645.

Flinn, C.J., 2002b. Interpreting minimum wage effects on wage distributions: a cautionary tale. Annales d'Économie et de Statistique 67/68, 309–355.

Flinn, C.J., Heckman, J.J., 1982. New methods for analyzing structural models of labor force dynamics. Journal of Econometrics 18, 115–168.

Fougère, D., Pradel, J., Roger, M., 1998. Public employment offices and the transition rate from unemployment to employment. Working paper. CREST-INSEE, Paris.

Friedman, M., 1962. Capitalism and Freedom. University of Chicago Press, Chicago.

Gilleskie, D.A., 1998. A dynamic stochastic model of medical care use and work absence. Econometrica 66, 1–45.

Heckman, J.J., 1981. Statistical models for discrete panel data. In: Manski, C.F., McFadden, D. (Eds.), Structural Analysis of Discrete Data with Econometric Applications. MIT Press, Cambridge.

Jovanovic, B., 1979. Job-matching and the theory of turnover. Journal of Political Economy 87, 972–990.

Karlin, S., 1962. Stochastic models and optimal policy for selling an asset. In: Arrow, K., et al. (Eds.), Studies in Applied Probability and Management Science. Stanford University Press, Stanford.

Katz, L., Autor, D., 1999. Changes in the wage structure and earnings inequality. In: Ashenfelter O., Card, D. (Eds.), Handbook of Labor Economics, vol. III. North-Holland, Amsterdam.

Keane, M., Wolpin, K., 1994. The solution and estimation of discrete choice dynamic programming models by simulation and interpolation: Monte Carlo evidence. Review of Economic and Statistics 76 (4), 648–672.

Keane, M., Wolpin, K., 1997. The career decisions of young man. Journal of Political Economy 105, 473–522.

Khandker, R.K., 1988. Offer heterogeneity in a two state model of sequential search. Review of Economics and Statistics 70, 259–265.

Kiefer, N., Neumann, G., 1979. An empirical job search model with a test of the constant reservation wage hypothesis. Journal of Political Economy 87, 69–82.

Kiefer, N.M., Neumann, G.R., 1993. Wage dispersion with homogeneity: the empirical equilibrium search model. In: Bunzel, H., et al. (Eds.), Panel Data and Labor Market Analysis. North Holland, Amsterdam.

Lancaster, T., 1990. The Econometric Analysis of Transition Data. Cambridge University Press, Cambridge.

Lancaster, T., Chesher, A., 1983. An econometric analysis of reservation wages. Econometrica 51, 1661–1677.

Ljungqvist, L., Sargent, T.J., 1998. The European unemployment dilemma. Journal of Political Economy 106, 514–550.

Lucas, R., Prescott, E., 1974. Equilibrium search and unemployment. Journal of Economic Theory 7, 188–209.

MacMinn, R.D., 1980. Job search and the labor dropout problem reconsidered. Quarterly Journal of Economics 95, 69–87.

McFadden, D., 1989. A method of simulated moments for estimation of discrete response models without numerical integration. Econometrica 57 (5), 995–1026.

Meyer, B.D., 1990. Unemployment insurance and unemployment spells. Econometrica 58, 757–782.

Mortensen, D., 1982. The matching process as a noncooperative bargaining game. In: McCall, J.J. (Ed.), The Economics of Information and Uncertainty. University of Chicago Press, Chicago.

Mortensen, D.T., 1986. Job search and labor market analysis. In: Ashenfelter, O., Layard, R. (Eds.), Handbook of Labor Economics. North-Holland, Amsterdam.

Mortensen, D.T., 1990. Equilibrium wage distributions: a synthesis. In: Hartog, J., et al. (Eds.), Panel Data and Labour Market Studies. North-Holland, Amsterdam.

Mortensen, D.T., 2003. Wage Dispersion: Why are Similar Workers Paid Differently? MIT Press, Cambridge.

Mortensen, D.T., Pissarides, C.A., 1999. New developments in models of search in the labor market. In: Ashenfelter, O., Card, D. (Eds.), Handbook of Labor Economics, vol. III. North-Holland, Amsterdam.

Petrongolo, B., Pissarides, C.A., 2001. Looking into the black box: a survey of the matching function. Journal of Economic Literature 39, 390–431.

Phelps, E.S., et al., 1970. Microeconomic Foundations of Employment and Inflation Theory. Norton, New York.

Pissarides, C.A., 1979. Job matching with state employment agencies and random search. Economic Journal 89, 818–833.

Pissarides, C.A., 1990. Equilibrium Unemployment Theory. Basil Blackwell, Oxford.

Postel-Vinay, F., Robin, J.M., 2002. Equilibrium wage dispersion with worker and employer heterogeneity. Econometrica 70, 2295–2350.

Ridder, G., van den Berg, G.J., 1997. Empirical equilibrium search models. In: Kreps, D.M., Wallis, K.F. (Eds.), Advances in Economics and Econometrics. Cambridge University Press, Cambridge.

Ridder, G., van den Berg, G.J., 2003. Measuring labor market frictions: a cross-country comparison. Journal of the European Economic Association 1, 224–244.

Rocheteau, G., Wright, R., 2005. Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. Econometrica 73, 175–202.

Rogerson, R., Shimer, R., Wright, R., 2005. Search-theoretic models of the labor market: a survey. Journal of Economic Literature 43, 959–988.

Rosenzweig, M.R., Wolpin, K.I., 2000. Natural "natural experiments" in economics. Journal of Economic Literature 38, 827–874.

Schoonbroodt, A., 2003. Small sample bias in a standard search model. Mimeo, University of Minnesota.

Stern, S., 1989. Estimating a simultaneous search model. Journal of Labor Economics 7, 348–369.

van den Berg, G.J., 1990. Nonstationarity in job search theory. Review of Economic Studies 57, 255–277.

van den Berg, G.J., 1999. Empirical inference with equilibrium search models of the labor market. Economic Journal 109, F283–F306.

van den Berg, G.J., 2001. Duration models: specification, identification, and multiple durations. In: Heckman, J.J., Leamer, E. (Eds.), Handbook of Econometrics, vol. V, North-Holland, Amsterdam.

van den Berg, G.J., Ridder, G., 1993a. On the estimation of equilibrium search models from panel data. In: van Ours, J.C., et al. (Eds.), Labor Demand and Equilibrium Wage Formation. North-Holland, Amsterdam.

van den Berg, G.J., Ridder, G., 1993b. Estimating an equilibrium search model from wage data. In: Bunzel, H., et al. (Eds.), Panel Data and Labor Market Analysis. North-Holland, Amsterdam.

van den Berg, G.J., Ridder, G., 1998. An empirical equilibrium search model of the labor market. Econometrica 66, 1183–1221.

van den Berg, G.J., van Vuuren, A., 2003. The effect of search frictions on wages. Working paper, Free University, Amsterdam.

Wolinsky, A., 1987. Matching, search and bargaining. Journal of Economic Theory 42, 311–333.

Wolpin, K.I., 1987. Estimating a structural job search model: the transition from school to work. Econometrica 55, 801–818.

Wolpin, K.I., 1992. The determinants of black–white differences in early employment careers: search, layoffs, quits and endogenous wage growth. Journal of Political Economy 100, 535–560.

Wolpin, K.I., 1995. Empirical methods for the study of labor force dynamics. Harwood Academic Publishers, Luxembourg.

Wolpin, K.I., 2003. Wage equations and education policy. In: Dewatripont, M., et al. (Eds.), Advances in Economics and Econometrics. Cambridge University Press, Cambridge.

Yoon, B.J., 1981. A model of unemployment duration with variable search intensity. Review of Economics and Statistics 63, 599–609.